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Water Management under Deficit Irrigation

Haqqi I. Yasin
haqqiismail56@gmail.com

Entesar M. Ghazal
entesarzal@gmail.com

Dams and Water Resources Engineering Departementand, College of Engineering, University of Mosul, Iraq

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ABSTRACT

Depending on the production function, irrigation water cost function and the sale price of yield, previously, lengthy mathematical expressions have been developed to estimate the optimal levels of water used in deficit irrigation, that would maximize yield (W_m), under water limiting (W_w), and under land limiting (W_l) and the amount of water that leads to income equal to income of W_m when land is limited (W_{el}), and that leads to farm income equal to farm income of W_m when water is limited W_{ew} . Therefore, in this paper, firstly the previous lengthy expressions for (W_{el}) and (W_{ew}) were simplified. Secondly, simple expressions for both (W_{el}) and (W_{ew}) were derived under presence of rainfall. Thirdly, very simple mathematical relations between (W_m & W_l) and (W_m & W_w) were derived in order to determine much easier expressions than those previously derived for W_{el}, and W_{ew} respectively, with and without rainfall.

Key words:
Deficit irrigation, Water management, Optimizing water use, Production functions.

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1. INTRODUCTION

The increasing use of irrigation water has led to a deterioration in water in many areas. In the race to boost agricultural productivity, irrigation will become more dependent on poorly developed and almost uncontrolled water sources [1]. Irrigated agriculture needs to contribute an increasing share of food production to meet the growing demands of the growing population. In the face of climate change impacts, many arid and semi-arid areas suffer from water shortages. Drought and increased water demand have led to the need for irrigation water management. Water stress negatively affects plants. Depending on the degree and extent of water deficit stress, plants growth and the physiological functions can be limited and affected [2], [3].

Water shortage is the main impediment of agricultural production, so agricultural research in arid and semi-arid areas is mainly focused on the relationship between crop production and water use. The challenge in agriculture is to increase the economic return from the lowest amount of water, which is either reducing water consumption with increasing production, so-called irrigation water management, or increase production with a fixed amount of water (agricultural management) that will eventually lead to more water production. Irrigation water management to increase production will not be the same in all areas. In areas where sufficient water is available, full irrigation can be an appropriate option while in areas where water is limited; irrigation is considered an appropriate strategy for more efficient use of resources available water.

The use of management strategies to achieve sustainable production as the economic dividend is an appropriate management. However, in all cases it is necessary to know the relationship between water and production (the productivity equation) in order to achieve sustainable production in optimum quantities of irrigation water. Crop production functions is a convenient tool for irrigation management and yield response estimation in Water scarcities [4].

A number of researchers have developed production functions in multiple forms, and the available equations can be used to derive and develop productivity equations. [5], [6], [7], [8].

The objective of this research is firstly, to simplify the lengthy equations that derived by (English, 1990 and Capra et al., 2008) for the optimum water level at which net income equals that at full irrigation when land is limited (W_{el}), and when water is limited (W_{ew}). Secondly, to derive expressions for both (W_{el}) and (W_{ew}) under presence of rainfall. Thirdly, the main objective is to derive easy to apply equations for both (W_{el}) and (W_{ew}) depending on irrigation water level that, maximizes yield per unit land (W_m), and that maximizes net income per unit land (W_l), and that maximizes net income per unit of water (W_w) which can be found with simplified equations and much easier than the previously derived equations.

2. THE PROPOSED METHODOLOGY

2.1. Conceptual model

By adopting the production function $y(w) = a + bw + cw^2$ (kg or $tonha^{-1}$) and irrigation water cost function of $C(w) = A + Bw$ ($\$ha^{-1}$) and the sale price of yield P, W is the amount of irrigation water, W_m, which represents the amount of irrigation water that will maximize yields, can be found by setting the derivative of production function to zero, $\boldsymbol{\partial Y(w)/\partial w=0}$.

Where y(w) = yield/unit land (kg or $tonha^{-1}$).

C(w) = cost/unit land ($\$ha^{-1}$).

a, b, c = production function constant.

A, B = irrigation water cost function constant.

P= sale price of yield ($\$kg^{-1}$ or $\$ton^{-1}$).

W = the amount of irrigation water ($m^3\ ha^{-1}$).

The net income function of applied water $il(w) = P * Y(w) - C(w)$($\$ha^{-1}$), the net farm income from all irrigated land ($) If(w)= il(w) $*$Area. When land is limited, the amount of irrigation water that gives maximum profit or income (w_l), can be found by setting the derivative of the income function to zero $\boldsymbol{\partial iL(w)/\partial w= 0}$. The amount of irrigation water that gives maximum farm income when water is limited (w_w) can be found by taking the derivative of the farm income function and set it to zero $\boldsymbol{\partial If(w)/\partial w=0}$.

w_{el} represents the amount of irrigation water that leads to profit or net income equal to the net income of W_m when land is limited **il(w_m)= il(w_{el}),** when irrigation water is limited W_{ew}, represents the amount of irrigation water that leads to profit or net farm income, equal to the net income of the farm when using Wm **If(w_m)= If(w_{ew}).** Figure (1) shows the quadratic production function and irrigation water cost function and W_m, W_l and W_{el}.

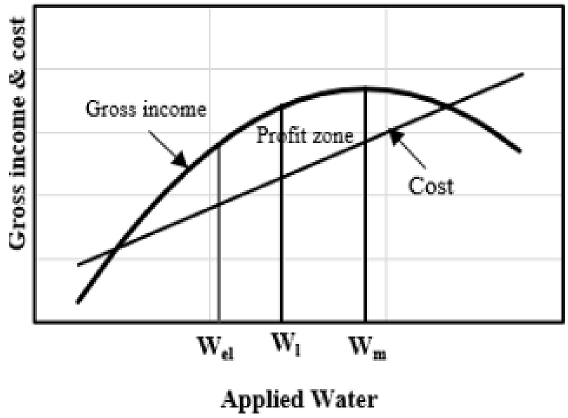

Figure 1: The quadratic production function, the irrigation water cost function, and W_m, W_l and W_{el}.

2.2 General mathematical composition

Using quadratic production function and linear cost function, [9] presented general expression of W_m, W_l and W_w and lengthy equations with two factors Z_1 and Z_2 to be compensated in equations 6A and 7A in order to determine W_{el} and W_{ew} respectively

$$Y(w) = a + bw + cw^2 \quad (1A)$$

$$C(w) = A + Bw \quad (2A)$$

$$W_m = -b/2c \quad (3A)$$

$$W_l = (B - bP)/(2Pc) \quad (4A)$$

$$W_w = ((Pa - A)/(Pc))^{0.5} \quad (5A)$$

$$W_{el} = \frac{B - Pb + Z_1}{2Pc} \quad (6A)$$

$$W_{ew} = (-Z_2 + [Z_2^2 - 4Pc(Pa - A]^{0.5})/(2Pc) \quad (7A)$$

Where

$$Z_1 = \left[(Pb - B)^2 - 4Pc\left(\frac{Pb^2}{4c} - \frac{bB}{2c}\right)\right]^{0.5}$$

$$Z_2 = \frac{Pb^2 - 4Ac + 4Pac}{2b}$$

Also [10] presented lengthy equations for W_{el} and W_{ew}. [11] gave full expressions of W_m, W_l and W_w with the quadratic production function and quadratic cost function and also presented lengthy equations for W_{el} and W_{ew} with two factors Z_1 and Z_2 as shown in the equations 1B-7B.

$$Y(w) = a_1 + b_1 w + c_1 w^2 \tag{1B}$$

$$C(w) = a_2 + b_2 w + c_2 w^2 \tag{2B}$$

$$W_m = -\frac{b_1}{2c_1} \tag{3B}$$

$$W_l = \frac{b_2 - Pb_1}{2(Pc_1 - c_2)} \tag{4B}$$

$$W_w = \left(\frac{(Pa_1 - a_2)}{Pc_1 - c_2}\right)^{0.5} \tag{5B}$$

$$W_{el} = (b_2 - Pb_1 + Z_1)/2(Pc_1 - c_2) \tag{6B}$$

$$W_{ew} = \frac{-Z_2 + [Z_2^2 - 4(Pa_1 - a_2)(Pc_1 - c_2]^{0.5}}{2(Pc_1 - c_2)} \tag{7B}$$

Where

$$Z_1 = \left[(Pb_1 - b_2)^2 - 4(Pc_1 - c_2)\left(\frac{b_{1(Pb_1 - b_2)}}{2c_1} - \frac{b^2_{1(Pc_1 - c_2}}{4c_1^2}\right)\right]^{0.5}$$

$$Z_2 = \frac{4(Pa_1 - a_2)c_1 - b_1^2(Pc_1 - c_2)}{2b_1 c_1}$$

(a_1, b_1, c_1), (a_2, b_2, c_2) = production and cost functions constants respectively.

In presence of rainfall, using quadratic production function and linear cost function [12] modified full expression for W_l, W_w [7]also presented full expressions for W_m, W_l, W_w but, without expressions for W_{el} and W_{ew}, as shown in the equations 1C-5C.

$$Y(w') = a' + b'w' + c'w'^2 \tag{1C}$$

$$C(w) = A + Bw \tag{2C}$$

$$W_m = -\frac{b'}{2c'} - R \tag{3C}$$

$$W_l = \frac{B - b'P}{2Pc'} - R \tag{4C}$$

$$W_w = ((P(a' + b'R + c'R^2) - A)/(Pc'))^{0.5} \tag{5C}$$

Where a′, b', c' = production function constants in presence of rainfall. R = rain. w′ = w+ R

3. RESULTS AND DISCUSSION

3.1. Current research derivation

As mentioned above the optimum water levels w_{el} and w_{ew} were derived previously but their derivation were complicated and lengthy. To achieve the main objective of our study, simple expressions for w_{el} and w_{ew} with different production and cost functions were very essential to derive simple mathematical relations between w_m, w_l, ww in order to determine w_{el} and w_{ew}. Therefore, firstly the lengthy equations that derived by [9] were simplified as shown in equations 8A, 9A respectively. As their derivations were complicated and lengthy, they were given in Annex A.

$$W_{el} = (2B - Pb)/(2Pc) \tag{8A}$$

$$W_{ew} = 2(A - Pa)/(bP) \tag{9A}$$

Using both quadratic production and cost functions, simple expressions were derived in compare to the lengthy equations that previously derived by [11] with two factors Z_1 and Z_2. As their full derivations were complicated and lengthy, they were given in Annex B.

$$W_{el} = \frac{2c_1 b_2 - b_1(Pc_1 + c_2)}{2c_1(Pc_1 - c_2)} \tag{8B}$$

$$W_{ew} = \frac{2c_1(a_2 - Pa_1)}{b_1(Pc_1 - c_2)} \tag{9B}$$

3.2. Derivation of rained condition

Under rained condition full expressions for W_{el} and W_{ew} were derived using quadratic production function and linear cost function as in equations 6C and 7C respectively and their full derivation are given in Annex C.

$$W_{el} = \frac{(2B - Pb')}{2Pc'} - R \tag{6C}$$

$$W_{ew} = \frac{2(A - P(a' + b'R + c'R^2)}{P(b' + 2c'R)} \tag{7C}$$

3.3. The relationship between W_m, W_l and W_{el}

The current study presented simple mathematical relations to determine w_{el} and w_{ew}

depending on w_m, w_l, ww in irrigation and rainfall cases, Simple expression was derived for W_{el} depending on $\mathbf{W_m}$, $\mathbf{W_l}$. From equations 3A and 8A as follow

$$w_{el} = B/Pc + w_m \qquad (10)$$

From equations 3A and 4A results:

$$w_l = B/Pc + w_m \qquad (11)$$

From equations 10 and 11 results simple relation to determine w_{el} as in equation (12).

$$\mathbf{w_{el} = 2w_l - w_m} \qquad \mathbf{(12)}$$

The comprehensiveness of this relationship can be verified by using the values of equations 3B and 4B in equation (12) to find the amount of equation 8B or to verify its application by using the values of equations 3C and 4C in equation (12) to find the amount of equation 6C, and figure (2) shows the relation between (w_{el}/w_m & w_l/w_m).

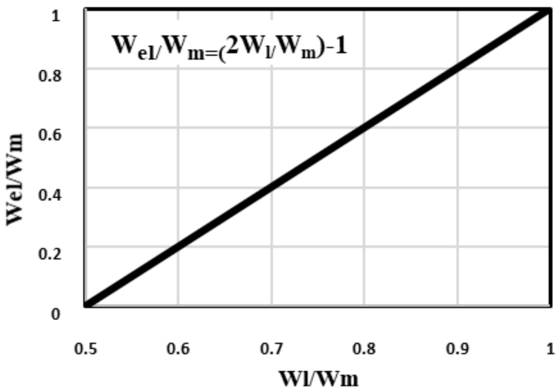

Figure (2): Relation between (w_{el}/w_m & w_l/w_m).

3.4. The relationship between W_m, W_w and W_{ew}

Simple expression was derived for W_{ew} depending on $\mathbf{W_m}$, $\mathbf{W_w}$. From equation 5A results

$$A = \left(\frac{a}{c} - W_w{}^2\right) P \qquad (13)$$

Compensation for the value of A (eq. 13) in equation, 9A produces:

$$w_{ew} = -2cw_w^2/b \qquad (14)$$

from equations 3A and 14 results

$$\mathbf{w_{ew} = w_w^2/w_m} \qquad \mathbf{(15)}$$

It can be verified from the comprehensiveness of this relationship in its application by using the values of equations 3B and 5B in equation (15) to find the magnitude of equation 9B or verification by using the values of equations 3C and 5C in equation (15) to find the amount of equation 7C. Figure (3) shows the relation between (w_{ew}/w_m & w_w/w_m).

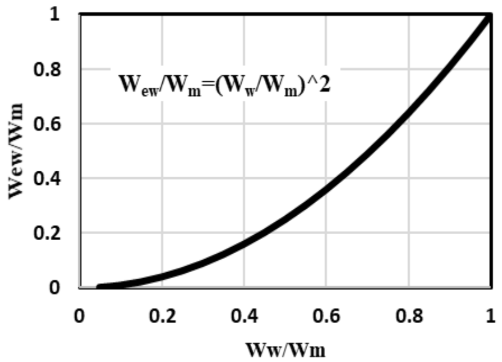

Figure (3): Relation between (w_{ew}/w_m& w_w/w_m).

4. CONCLUSION

It is easy to fully express the amounts of irrigation water referred to above W_m ,W_l , W_w , regardless to the formula of both the productivity function or the cost function, but there is a great difficulty in fully expressing each of the water quantities W_{el} and W_{ew} **.** So the research presented equations (12) and (15) or figures (2) and (3) where through them and amounts of water W_{el} and W_{ew} can be easily found depending on the amount of water W_m ,W_l ,W_w.

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Annex A

$$y(w) = a + bw + cw^2 \quad ; \quad C(w) = A + Bw$$

$$\frac{\partial y(w)}{\partial w} = b + 2cw = 0 \quad ; \quad w_m = \frac{-b}{2c}$$

Deriving W$_{el}$: $il(w_m) = P * Y(w_m) - C(w_m)$, $\quad il(w_{el}) = P * (w_{el}) - C(w_{el})$

$$il(w_m) = P * (a + bw_m + cw_m^2) - (A + Bw_m) = P * \left(a - \frac{b^2}{2c} + \frac{cb^2}{4c^2}\right) - A + \frac{bB}{2*c}$$

$$il(w_{el}) = P * (a + bw_{el} + cw_{el}^2) - (A + Bw_{el})$$

$$il(w_m) = il(w_{el}) \ , \quad Pa - \frac{Pb^2}{4c} - A + \frac{bB}{2c} = Pa + Pbw_{el} + Pc\, w_{el}{}^2 - A - Bw_{el}$$

$$Pcw_{el}{}^2 + (Pb - B)\, w_{el} + \frac{Pb^2}{4c} - \frac{bB}{2c} = 0$$

$$W_{el} = \frac{-Pb + B \pm \sqrt{P^2b^2 - 2PbB + B^2 - 4Pc\, \frac{Pb^2}{4c} + 4Pc\frac{bB}{2c}}}{2Pc}$$

$$W_{el} = \frac{-Pb + B \pm \sqrt{P^2b^2 - 2PbB + B^2 - P^2b^2 + 2PbB}}{2Pc}$$

$$\mathbf{W_{el} = \frac{2B - Pb}{2Pc}} \qquad \textbf{(8A)}$$

Deriving W$_{ew}$:

$$If(w_m) = (P * y(w_m) - C(w_m)) * Area \ , \quad Area = \frac{WT}{w_m} \quad , \quad w_m = \frac{-b}{2c}$$

$$\mathrm{If(w_m)} = \frac{\mathrm{WT}}{\mathrm{w_m}} * [\mathrm{P} * (\mathrm{a} + \mathrm{bw_m} + \mathrm{cw_m}^2) - (\mathrm{A} + \mathrm{Bw_m}] \;\; = \frac{-2\mathrm{cWT}}{\mathrm{b}}\left[\mathrm{Pa} - \frac{\mathrm{Pb^2}}{2\mathrm{c}} + \frac{\mathrm{Pb^2}}{4\mathrm{c}} - \mathrm{A} + \frac{\mathrm{bB}}{2\mathrm{c}}\right]$$

$$\mathrm{If(w_m)} = \mathrm{WT} * \left[\frac{-2Pac}{\mathrm{b}} + \frac{bP}{2} + \frac{2cA}{\mathrm{b}} - B\right]$$

$$\mathrm{If(w_{ew})} = \frac{\mathrm{WT}}{w_{ew}} * [\mathrm{Pa} + \mathrm{Pbw_{ew}} + \mathrm{Pcw_{em}}^2 - \mathrm{A} - \mathrm{Bw_{ew}}]\,, \quad \mathrm{If(w_m)} = \mathrm{If(w_{ew})}$$

$$\mathrm{WT} * \left[\frac{-2Pac}{\mathrm{b}} + \frac{bP}{2} + \frac{2cA}{\mathrm{b}} - B\right] = \frac{\mathrm{WT}}{w_{ew}} * [\mathrm{Pa} + \mathrm{Pbw_{ew}} + \mathrm{Pcw_{em}}^2 - \mathrm{A} - \mathrm{Bw_{ew}}]$$

$$\frac{-2\mathrm{Pac}}{\mathrm{b}} \mathrm{w_{ew}} + \frac{\mathrm{bP}}{2} \mathrm{w_{ew}} + \frac{2\mathrm{cA}}{\mathrm{b}} \mathrm{w_{ew}} - \mathrm{B\,w_{ew}} = \mathrm{Pa} + \mathrm{Pbw_{ew}} + \mathrm{Pc\,w_{ew}}^2 - \mathrm{A} - \mathrm{B\,wew}$$

$$+\mathrm{Pcw_{em}}^2 + \left[\frac{\mathrm{bP}}{2} - \frac{2\mathrm{cA}}{\mathrm{b}} + \frac{2\mathrm{Pac}}{\mathrm{b}}\right] \mathrm{w_{ew}} + \mathrm{Pa} - \mathrm{A} = 0 \quad \ldots\ldots\ldots..* \,(2b)$$

$$2\mathrm{bPcw_{ew}}^2 - \; [\mathrm{Pb^2} - 4\mathrm{cA} + 4\mathrm{Pac}]\mathrm{w_{ew}} + 2\mathrm{Pba} - 2\mathrm{bA} = 0$$

$$\mathrm{w_{ew}} = \frac{-(\mathrm{Pb^2} - 4\mathrm{cA} + 4\mathrm{Pac}) \pm \sqrt{\mathrm{P^2b^4} - 8\mathrm{P_cb^2\,cA} + 16\mathrm{c^2\,A^2} + 8\mathrm{P^2b^2ac} - 32\mathrm{Pac^2\,A} + 16\mathrm{P^2a^2c^2} - 16\mathrm{P^2b^2ac} + 16\mathrm{Pb^2cA}}}{4\mathrm{bPc}}$$

$$\mathrm{w_{ew}} = \frac{-(\mathrm{Pb^2}-4\mathrm{cA}+4\mathrm{Pac}) \pm \sqrt{(\mathrm{Pb^2}+4\mathrm{cA}-4\mathrm{Pac})^2}}{4\mathrm{bPc}} \qquad \textbf{-ve} \qquad \mathrm{w_m} = \frac{-\mathrm{b}}{2\mathrm{c}}$$

$$\mathbf{w_{ew}} = \frac{\mathbf{2A - 2Pa}}{\mathbf{bP}} \qquad \mathbf{(9A)}$$

Annex B

Deriving W$_{el}$: $\mathrm{Y(w)} = \mathrm{a_1} + \mathrm{b_1w} + \mathrm{c_1w^2}$, $\mathrm{C(w)} = \mathrm{a_2} + \mathrm{b_2w} + \mathrm{c_2w^2}$, $\mathrm{w_m} = \frac{-\mathrm{b_1}}{2\mathrm{c_1}}$

$$\mathrm{il(w_m)} = \mathrm{P} * \mathrm{Y(w_m)} - \mathrm{C(w_m)} \; = \mathrm{P} * (\mathrm{a_1} + \mathrm{b_1w_m} + \mathrm{c_1w_m^2}) - (\mathrm{a_2} + \mathrm{b_2w_m} + \mathrm{c_2})\mathrm{w_m^2}$$

$$\mathrm{il(w_m)} = \mathrm{P}a_1 + \mathrm{P}b_1\mathrm{w_m} + \mathrm{P}c_1\mathrm{w_m^2} - \mathrm{a_2} - \mathrm{b_2w_m} - \mathrm{c_2w_m^2}$$

$$\mathrm{il(w_m)} = \mathrm{P}a_1 + \mathrm{P}b_1\left(\frac{-\mathrm{b_1}}{2\mathrm{c_1}}\right) + \mathrm{Pc_1}\left(\frac{\mathrm{b_1^2}}{4\mathrm{c_1^2}}\right) - \mathrm{a_2} - \mathrm{b_2}\left(\frac{-\mathrm{b_1}}{2\mathrm{c_1}}\right) - \mathrm{c_2}\left(\frac{\mathrm{b_1^2}}{4\mathrm{c_1^2}}\right)$$

$$\mathrm{il(w_m)} = \mathrm{pa_1} - \frac{\mathrm{pb_1}^2}{2\mathrm{c_1}} + \frac{\mathrm{pb_1}^2}{4\,\mathrm{c_1}} - \mathrm{a_2} + \frac{\mathrm{b_2b_1}}{2\mathrm{c_1}} - \frac{\mathrm{c_2b_1}^2}{4\mathrm{c_1}^2} = \mathrm{pa_1} - \frac{\mathrm{pb_1}^2}{4\mathrm{c_1}} - \mathrm{a_2} + \frac{\mathrm{b_2b_1}}{2\mathrm{c_1}} - \frac{\mathrm{c_2b_1}^2}{4\mathrm{c_1}^2}$$

$$\mathrm{il(w_{el})} = \mathrm{pa_1} + \mathrm{pb_1w_{el}} + \mathrm{pc_1w_{el}}^2 - \mathrm{a_2} - \mathrm{b_2w_{el}} - \mathrm{c_2w_{el}}^2\,, \quad \mathrm{il(w_m)} = \mathrm{il(w_{el})}$$

$$\mathrm{pa_1} - \frac{\mathrm{pb_1}^2}{4\mathrm{c_1}} - \mathrm{a_2} + \frac{\mathrm{b_2b_1}}{2\mathrm{c_1}} - \frac{\mathrm{c_2b_1}^2}{4\mathrm{c_1}^2} = \mathrm{pa_1} + \mathrm{pb_1w_{el}} + \mathrm{pc_1w_{el}}^2 - \mathrm{a_2} - \mathrm{b_2w_{el}} - \mathrm{c_2w_{el}}^2$$

$$\mathrm{w_{el}}^2(\mathrm{pc_1} - \mathrm{c_2}) + \mathrm{w_{el}}(\mathrm{pb_1} - \mathrm{b_2}) + -\frac{\mathrm{pb_1}^2}{4\mathrm{c_1}} - \frac{\mathrm{b_2b_1}}{2\mathrm{c_1}} + \frac{\mathrm{c_2b_1}^2}{4\mathrm{c_1}^2} = 0$$

$$\mathrm{w_{el}} = \frac{-(p\mathrm{b_1} - \mathrm{b_2}) \pm \sqrt{\mathrm{P^2b_1}^2 - 2p\mathrm{b_2b_1} + \mathrm{b_2}^2 - \mathrm{P^2b_1}^2 + 2p\mathrm{b_2b_1} - \frac{p\mathrm{c_2b_1}^2}{\mathrm{c_1}} + \frac{p\mathrm{c_2b_1}^2}{\mathrm{c_1}} - \frac{2\mathrm{c_2b_2b_1}}{\mathrm{c_1}} + \frac{\mathrm{c_2}^2\mathrm{b_1}^2}{\mathrm{c_1}^2}}}{2(p\mathrm{c_1} - \mathrm{c_2})}$$

$$\mathrm{w_{el}} = \frac{-(p\mathrm{b_1}-\mathrm{b_2}) \pm \sqrt{(\mathrm{b_2} - \frac{\mathrm{c_2b_1}}{\mathrm{c_1}})^2}}{2(p\mathrm{c_1}-\mathrm{c_2})} \quad , \quad \text{-ve} \; = \; \mathrm{w_m} = \frac{-\mathrm{b_1}}{2\mathrm{c_1}}$$

$$\mathbf{w_{el}} = \frac{\mathbf{2c_1b_2 - b_1(\boldsymbol{p}c_1 + c_2)}}{\mathbf{2c_1(\boldsymbol{p}c_1 - c_2)}} \qquad \mathbf{(8\boldsymbol{B})}$$

Deriving W$_{ew}$:

$$\mathrm{If(w_m)} = (\mathrm{P} * \mathrm{y(w_m)} - \mathrm{C(w_m)}) * \mathrm{Area} = \frac{\mathrm{WT}}{\mathrm{w_m}} * [\mathrm{P} * (\mathrm{a_1} + \mathrm{b_1w_m} + \mathrm{c_1w_m^2}) - (\mathrm{a_2} + \mathrm{b_2w_m} + \mathrm{c_2})\mathrm{w_m^2}]$$

$$\mathrm{If}(w_m) = \frac{WT}{w_m} * [Pa_1 + Pb_1 w_m + Pc_1 w_m^2 - a_2 - b_2 w_m - c_2 w_m^2]$$

$$\mathrm{If}(w_m) = \frac{-2WTc_1}{b_1}\left[pa_1 - \frac{pb_1{}^2}{2c_1} + \frac{pb_1{}^2}{4\,c_1} - a_2 + \frac{b_2 b_1}{2c_1} - \frac{c_2 b_1{}^2}{4c_1{}^2}\right] = \frac{-2WTc_1}{b_1}\left[pa_1 - \frac{pb_1{}^2}{4c_1} - a_2 + \frac{b_2 b_1}{2c_1} - \frac{c_2 b_1{}^2}{4c_1{}^2}\right]$$

$$\mathrm{If}(w_{ew}) = \frac{WT}{w_{ew}}[pa_1 + pb_1 w_{ew} + pc_1 w_{ew}{}^2 - a_2 - b_2 w_{ew} - c_2 w_{ew}{}^2]\ ,\ \mathrm{If}(w_m) = \mathrm{If}(w_{ew})$$

$$pa_1 + pb_1 w_{ew} + pc_1 w_{ew}{}^2 - a_2 - b_2 w_{ew} - c_2 w_{ew}{}^2 + \frac{2pa_1 c_1 w_{ew}}{b_1} - \frac{pb_1 w_{ew}}{2} - \frac{2c_1 a_2 w_{ew}}{b_1} + b_2 w_{ew} - \frac{c_2 b_1 w_{ew}}{c_1} = 0$$

$$w_{ew}{}^2(pc_1 - c_2) + w_{ew}\left(\frac{pb_1}{2} + \frac{2pa_1c_1}{b_1} - \frac{2\,a_2c_1}{b_1} - \frac{c_2b_1}{2c_1}\right) + pa_1 - a_2 = 0$$

$$w_{ew} = \frac{-\left(\frac{pb_1}{2} + \frac{2pa_1c_1}{b_1} - \frac{2\,a_2c_1}{b_1} - \frac{c_2b_1}{2c_1}\right) \pm \sqrt{\frac{b_1{}^2p^2}{4} + p^2a_1c_1 - p\,a_2c_1 - \frac{pc_2b_1{}^2}{4\,c_1} + \frac{4\,a_1{}^2p^2c_1{}^2}{b_1{}^2} + p^2a_1c_1 - \frac{4pa_1a_2c_1{}^2}{b_1{}^2} - p\,a_1c_2 + \frac{4\,a_2{}^2c_1{}^2}{b_1{}^2}}}{2(pc_1 - c_2)}$$

$$\ldots\ldots\sqrt{Pa_2c_1 - \frac{4pa_1a_2c_1{}^2}{b_1{}^2} + a_2c_2 + \frac{c_2{}^2b_1{}^2}{4c_1{}^2} - \frac{pc_2b_1{}^2}{4\,c_1} - p\,a_1c_2 + a_2c_2 - 4p^2a_1c_1 + 4p\,a_2c_1 + 4p\,a_1c_2 - 4\,a_2c_2}$$

$$w_{ew} = \frac{-(\frac{pb_1}{2} + \frac{2pa_1c_1}{b_1} - \frac{2\,a_2c_1}{b_1} - \frac{c_2b_1}{2c_1}) \pm \sqrt{(\frac{pb_1}{2} - \frac{2pa_1c_1}{b_1} + \frac{2\,a_2c_1}{b_1} - \frac{c_2b_1}{2c_1})^2}}{2(pc_1 - c_2)}$$

$$w_{ew} = \frac{-\frac{pb_1}{2} - \frac{2pa_1c_1}{b_1} + \frac{2\,a_2c_1}{b_1} + \frac{c_2b_1}{2c_1} \pm (\frac{pb_1}{2} - \frac{2pa_1c_1}{b_1} + \frac{2\,a_2c_1}{b_1} - \frac{c_2b_1}{2c_1})}{2(pc_1 - c_2)}\quad , \quad \text{-ve} \quad , \quad w_m = \frac{-b_1}{2c_1}$$

$$w_{ew} = \frac{-\frac{pb_1}{2} - \frac{2pa_1c_1}{b_1} + \frac{2\,a_2c_1}{b_1} + \frac{c_2b_1}{2c_1} - \frac{pb_1}{2} + \frac{2pa_1c_1}{b_1} - \frac{2\,a_2c_1}{b_1} + \frac{c_2b_1}{2c_1})}{2(pc_1 - c_2)} = \frac{-\frac{4pa_1c_1}{b_1} + \frac{4\,a_2c_1}{b_1}}{2(pc_1 - c_2)}$$

$$\mathbf{w_{ew}} = \frac{\mathbf{2c_1(a_2 - Pa_1)}}{\mathbf{b_1(Pc_1 - c_2)}} \qquad \mathbf{(9B)}$$

Annex C

Deriving W$_{el}$

$Y\,(w') = a' + b'\,w' + c'\,w'^2$; $w' = (w + R)$, $C(w) = A + Bw$

$Y\,(w + R) = a' + b'\,(w + R) + c'\,(w + R)^2 = a' + b'\,w + b'\,R + c'\,w^2 + 2c'wR + c'R^2$

$$\frac{\partial y(w+R)}{\partial w} = b' + 2c'R + 2c'w = 0 \quad , \quad w_m = \frac{-b'}{2c'} - R$$

$$\mathrm{il}(w_m) = P * Y(w_m) - C(w_m)$$

$$\mathrm{il}(w_m) = P * (a' + b'\,w_m + b'\,R + c'\,w_m^2 + 2c'\,w_m R + c'R^2) - (A + Bw_m)$$

$$\mathrm{il}(w_m) = P * \left[a' + b'\,R + c'R^2 + (b' + 2c'R)\left(\frac{-b'}{2c'} - R\right) + c'(\frac{-b'}{2c'} - R)^2\right] - \left(A + B(\frac{-b'}{2c'} - R)\right)$$

$$\mathrm{il}(w_m) = P\,a' + Pb'R + Pc'R^2 - \frac{Pb'^2}{2c'} - Pb'R - Pb'R - Pb'R^2 - 2Pc'R^2 + \frac{Pb'^2}{4c'} + Pb'R + Pc'R^2 - A + \frac{Bb'}{2c'} + BR$$

$$\mathrm{il}(w_m) = P\,a' - \frac{Pb'^2}{4c'} + \frac{Bb'}{2c'} - A + BR$$

$$\mathrm{il}(\mathrm{w_{el}}) = P\,\mathrm{a'} + \mathrm{Pb'R} + \mathrm{Pc'R^2} + P(\mathrm{b'} + 2\mathrm{c'R})w_{el} + P\mathrm{c'w_{el}}^2 - A - Bw_{el}\,, \qquad \mathrm{il}(\mathrm{w_m}) = \mathrm{il}(\mathrm{w_{el}})$$

$$P\,\mathrm{a'} - \frac{\mathrm{Pb'}^2}{4\mathrm{c'}} + \frac{B\mathrm{b'}}{2\mathrm{c'}} - A + BR = P\,\mathrm{a'} + \mathrm{Pb'R} + \mathrm{Pc'R^2} + P(\mathrm{b'} + 2\mathrm{c'R})w_{el} + P\mathrm{c'w_{el}}^2 - A - Bw_{el}$$

$$P\mathrm{c'w_{el}}^2 + (P\mathrm{b'} + 2\mathrm{Pc'R} + \mathrm{B})w_{el} + \mathrm{Pb'R} + \mathrm{Pc'R^2} + \frac{P\mathrm{b'}^2}{4\mathrm{c'}} - \frac{B\mathrm{b'}}{2\mathrm{c'}} - BR = 0$$

$$\mathrm{w_{el}} = \frac{-(\mathrm{Pb'} + 2\mathrm{Pc'R} + \mathrm{B} \pm \sqrt{\mathrm{P^2b'^2} + 4\mathrm{P^2\,b'Rc'} + 4\mathrm{P^2R^2c'^2} - 2\mathrm{Pb'B} - 4\mathrm{PBc'R} + \mathrm{B^2} - 4\mathrm{P^2\,b'Rc'} - 4\mathrm{P^2R^2c'^2} - \mathrm{P^2b'^2} + 2\mathrm{Pb'B} + 4\mathrm{PBc'R}}}{2\mathrm{Pc'}}$$

$$\mathbf{w_{el}} = \frac{\mathbf{2B - Pb'}}{\mathbf{2Pc'}} - \mathbf{R} \qquad \mathbf{(6C)}$$

Deriving $\mathbf{W_{ew}}$:

$$\mathrm{If}(\mathrm{w_m}) = \mathrm{il}(\mathrm{w_m}) * \mathrm{Area}\,, \qquad \mathrm{Area} = \frac{WT}{w_m} \qquad , \qquad \mathrm{w_m} = \frac{-\mathrm{b'}}{2\mathrm{c'}} - \mathrm{R}$$

$$\mathrm{If}(\mathrm{W_m}) = \mathrm{il}(\mathrm{W_m}) * \mathrm{Area} = [\mathrm{P\{a + b(W_m + R) + c(W_m + R)^2\} - A - BW_m}] * \frac{\mathrm{WT}}{\mathrm{W_m}}$$

$$\mathrm{If}(\mathrm{W_m}) = \frac{\mathrm{WT}}{\mathrm{W_m}}[\mathrm{P(a' + b'R + c'R^2) + Pb'W_m + Pc'W_m^2 + 2Pc'RW_m - A - BW_m}]$$

$$\mathrm{If}(\mathrm{W_m}) = \mathrm{WT}\left[\frac{\mathrm{P(a'+b'R+c'R^2)}}{\mathrm{W_m}} + \mathrm{b'P} + \mathrm{Pc'W_m} + 2\mathrm{Pc'R} - \frac{\mathrm{A}}{\mathrm{W_m}} - \mathrm{B}\right],$$

$$\mathrm{If}(\mathrm{W_{ew}}) = \mathrm{WT}\left[\frac{\mathrm{P(a'+b'R+c'R^2)}}{\mathrm{W_{ew}}} + \mathrm{b'P} + \mathrm{Pc'W_{ew}} + 2\mathrm{Pc'R} - \frac{\mathrm{A}}{\mathrm{W_{ew}}} - \mathrm{B}\right], \qquad \mathrm{If}(\mathrm{w_m}) = \mathrm{If}(\mathrm{w_{ew}})$$

$$\frac{\mathrm{P(a' + b'R + c'R^2)}}{\mathrm{W_m}} + \mathrm{Pc'W_m} - \frac{\mathrm{A}}{\mathrm{W_m}} = \frac{\mathrm{P(a' + b'R + c'R^2)}}{\mathrm{W_{ew}}} + \mathrm{Pc'W_{ew}} - \frac{\mathrm{A}}{\mathrm{W_{ew}}}$$

$$\mathrm{PZW_{ew} + Pc'W_m^2.W_{ew} - AW_{ew} = PZW_m + Pc'W_mW_{ew}^2 - AW_m}$$

$$\mathrm{Pc'W_mW_{ew}^2 - (PZ + Pc'W_m^2 - A)W_{ew} + PZW_m - AW_m = 0}$$

$$\mathrm{W_{ew}} = \frac{\mathrm{PZ + Pc'W_m^2 - A} \mp \sqrt{\mathrm{(PZ + Pc'W_m^2)^2 - 2A(PZ + Pc'W_m^2) + A^2 - 4Pc'W_m(PZW_m - AW_m)}}}{\mathrm{2Pc'W_m}}$$

$$= \frac{\mathrm{PZ + Pc'W_m^2 - A} \mp \sqrt{\mathrm{P^2Z^2 + 2PZPc'W_m^2 + P^2c'^2W_m^4 - 2APZ - 2APc'W_m^2 + A^2 - 4P^2c'ZW_m^2 + 4Pc'AW_m^2}}}{\mathrm{2Pc'W_m}}$$

$$\mathrm{W_{ew}} = \frac{\mathrm{PZ + Pc'W_m^2 - A} \mp \sqrt{\mathrm{(PZ - Pc'W_m^2)^2 - 2APZ + 2Pc'AW_m^2 + A^2}}}{\mathrm{2Pc'W_m}}$$

$$\mathrm{W_{ew}} = \frac{\mathrm{PZ + Pc'W_m^2 - A} \mp \sqrt{\mathrm{(PZ - Pc'W_m^2)^2 - 2A(PZ - Pc'W_m^2) + A^2}}}{\mathrm{2Pc'W_m}}$$

$$\mathrm{W_{ew}} = \frac{\mathrm{PZ + Pc'W_m^2 - A} \mp \sqrt{\mathrm{(PZ - Pc'W_m^2 - A)^2}}}{\mathrm{2Pc'W_m}} = \frac{\mathrm{PZ + Pc'W_m^2 - A} \mp \mathrm{(PZ - Pc'W_m^2 - A)}}{\mathrm{2Pc'W_m}}$$

$$\mathrm{W_{ew}} = \frac{\mathrm{PZ + Pc'W_m^2 - A + PZ - Pc'W_m^2 - A}}{\mathrm{2Pc'W_m}} = \frac{\mathrm{2(PZ - A)}}{\mathrm{2Pc'W_m}} = \frac{\mathrm{[P(a' + b'R + c'R^2) - A]}}{\mathrm{Pc'}\left[\frac{-\mathrm{b'}}{2\mathrm{c'}} - \mathrm{R}\right]}$$

$$\mathbf{W_{ew}} = \frac{\mathbf{2[A - P(a' + b'R + c'R^2)]}}{\mathbf{P[b' + 2c'R]}} \qquad \mathbf{(7C)}$$

إدارة المياه تحت الري الناقص

د. حقي إسماعيل ياسين
haqqiismail56@gmail.com

انتصار محمد غزال
entesarzal@gmail.com

جامعة الموصل - كلية الهندسة - قسم هندسة السدود والموارد المائية

الملخص

اعتمادا على معادلة الانتاجية، وكلفة مياه الري وسعر بيع الغلة، تم وضع مجموعة تعابير رياضيتيه مطولة لتقدير المستويات المثلى للري الناقص الذي من شأنه تعظيم الغلة(Wm) *عند محدودية مياه الري*(Ww) *ومحدودية الأراضي الزراعية* (Wl)*، كذلك كمية المياه التي تنتج غلة تعادل اقصى غلة متوقعة للحقل عندما تكون الأرض محددة* (Wel)*، وعندما تكون كمية المياه محددة* (Wew)*. تم في البحث الحالي، أولا تبسيط المعادلات الرياضياتية المطولة لكل من* (Wel)*،*(Wew) *، ثانيا اشتقاق تعابير مبسطة لكل من* (Wel)*،* (Wew)*لحالة وجود المطر، ثالثا تم اشتقاق علاقات رياضياتية بسيطة جدا تربط بين* (Wm & Wl) *و* (Wm & Ww) *والتي تعتبر سهلة القياس، لغرض ايجاد تعابير أسهل بكثير من تلك المشتقة سابقا لكل من* (Wel)*،*(Wew) *ولحالتي وجود وغياب الامطار.*

الكلمات الداله:

الري الناقص، ادارة المياه، معادلة الإنتاجية، أمثلية استخدام المياه.